

Theoretical signal-to-noise ratio of a slotted surface coil for magnetic resonance imaging

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Abstract

The analytical expression for the signal-to-noise ratio of a slotted surface coil with an arbitrary number of slots was derived using the quasi-static approach. This surface coil based on the vane-type magnetron tube. To study the coil performance, the theoretical signal-to-noise ratio predictions of this coil design were computed using a different number of slots. Results were also compared with theoretical results obtained for a circular coil with similar dimensions. It can be appreciated that slotted surface coil performance improves as the number of coils increases and, outperformed the circular-shaped coil. This makes it a good candidate for other MRI applications involving coil array techniques.

1 Introduction

Radio-frequency (RF) coils constitute the key hardware component for the transmission and reception of the magnetic resonance signal. The performance characteristics are a crucial element in the determination of image quality as measured by the signal-to-noise ratio (*SNR*), signal homogeneity, and spatial resolution. The *SNR* is the widely-accepted parameter to measure coil performance since is independent of the imaging parameters and the signal processing system. A great deal of effort has been done to develop RF coils for different Magnetic Resonance Imaging (MRI) and Magnetic Resonance Spectroscopy (MRS) applications since the publication of the seminal

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papers by Hoult, Richard and Lauterbur [1-2]. MRI scientists have proposed different theoretical approaches to derive expressions for the *SNR* models involving low and high frequency approaches [3-9]. To find an analytical solution even for simple coil geometries requires the use of an intrinsically-difficult mathematical frame. This has motivated somehow to propose numerical solutions as an alternative method [10-11].

The research work presented here was motivated by the renewed interest in the development of RF surface coils caused by the introduction of parallel imaging (PI) [12]. Parallel imaging requires of coil arrays with high performance, however the study of the individual coil performance plays an important role to achieve a high *SNR* of the coil array [13-14].

A coil design based on the cavity magnetron tube [15-16] was introduced by Rodriguez [17], which showed a coil performance improvement over the circular coil. An interesting historical fact, it is that a magnetron tube was also used in the early NMR experiments to detect the NMR signal by Purcell, Torrey and Pound back in 1945 [15]. These experimental results obtained with the magnetron surface coil stimulated used to calculate an *SNR* formula to guide the further development of this type of coils on a reliable manner.

The objective of this work was to develop a theoretical model of the signal-to-noise ratio for a the vane-type magnetron surface coil design and named the slotted surface coil, see Fig. 1.

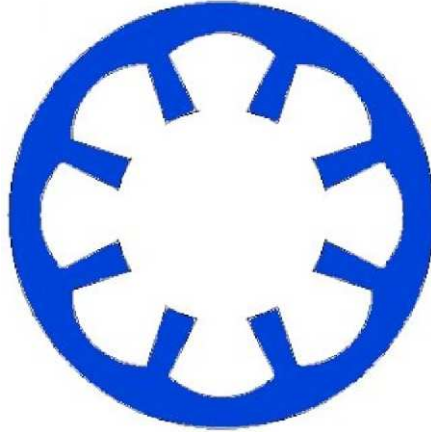


Figure 1. This illustration shows the vane type configuration.

Then, a general *SNR* expression was derived using the quasi-static approach as a function of the number of vanes (slots). The formula was used to investigate the performance of this coil design varying the number of slots used for particular coil dimensions. It also served to theoretically compare the coil performance against a circular-shaped coil with similar dimensions for fair comparison.

2 Signal-to-Noise Ratio (*SNR*)

The signal-to-noise ratio determines the performance of a surface coil, which is proportional to the ratio of the induced MR-signal to the root-mean-square (rms) of the thermal noise voltage at the coil terminals and the sample. According to Richard and Hoult [1], the *SNR* can then be expressed

$$SNR(r) = \frac{\omega V M B_1(r)}{\sqrt{8kT\Delta f(P_A + P_B)}} \quad (1)$$

where ω is the Larmor frequency, V is the sample volume, $B_1(r)$ is the magnetic field at position r (Figure 2), k is Boltzmann's constant, T is the absolute temperature, Δf is the receiver bandwidth, P_A and P_B are the power loss within the coil and the sample, respectively, when the coil carries a given current I or, more general, a given current distribution J . M is the magnetisation perpendicular to the static magnetic field and determined by the RF pulse sequence [3]. To evaluate the SNR of the slotted coil is necessary to compute both the magnetic field and the power losses in Eq. (1).

2.1 Calculation of Magnetic Field $B_1(r)$

According to the quasi-static approach and Fig. 2, the magnetic field $B_1(r)$ in Cartesian coordinates can be obtained from the Biot-Savart law,

$$B_1(r) = \frac{\mu_0}{4\pi} \int_{S'} \frac{J(r') \times (r - r') dS'}{|r - r'|^3} = \frac{\mu_0}{4\pi} \int_{S'} \frac{(J_y z) e_x + (-J_x z) e_y + (-J_x y' + J_y x') e_z dS'}{(x'^2 + y'^2 + z^2)^{3/2}} \quad (2)$$

where $J(r') = J_x e_x + J_y e_y$ is the current density and $r - r' = (-x') e_x + (-y') e_y + z e_z$. Finally, r is the observation point and r' is the position of area element and μ_0 is the permeability.

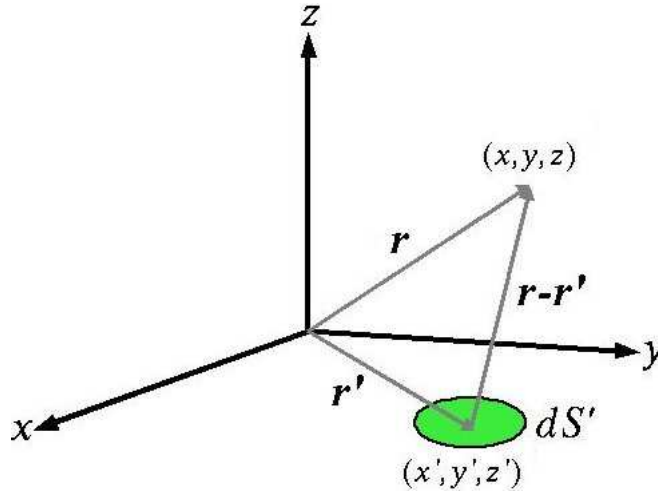


Figure 2. Schematic of geometric arrangement for the computation of B_1 at observation point, (x', y', z') created by an infinitesimal coil located at point, $r(x, y, z)$

From the coil geometry it is suggested to express Eq. (2) in cylindrical coordinates

$$B_1(z) = \frac{\mu_0}{4\pi} \left[\begin{aligned} & \int_{S'} \frac{J_\phi z (\cos(\phi) \cos(\phi') + \sin(\phi) \sin(\phi')) dS'}{(\rho'^2 + z^2)^{3/2}} e_\rho \\ & + \int_{S'} \frac{J_\phi z (\cos(\phi) \sin(\phi') - \sin(\phi) \cos(\phi')) dS'}{(\rho'^2 + z^2)^{3/2}} e_\phi \\ & + \int_{S'} \frac{J_\phi \rho' dS'}{(\rho'^2 + z^2)^{3/2}} e_z \end{aligned} \right] \quad (3)$$

If it is assumed that $dS' = \rho' d\rho' d\phi'$ the z -component of the magnetic field in Eq. (3) is

$$B_1(z) = \frac{\mu_0}{4\pi} \int_{\phi' \in \Re_m} \frac{J_\phi \rho'^2 d\rho' d\phi'}{(\rho'^2 + z^2)^{3/2}} e_z \quad (4)$$

where \Re_m is the integration area in Fig. 3.

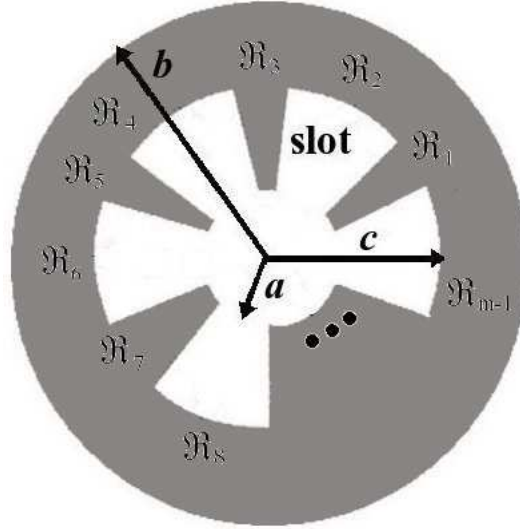


Figure 3. Diagram of the slotted surface coil and design parameters showing the area of integration.

The linearly polarized magnetic field of the slotted surface coil can be computed similarly as in [19],

$$B_1(z) = \frac{\mu_0 I n}{2\pi} \left(\begin{aligned} & \frac{(\frac{\pi}{n} - \phi_0)}{(b-a)} \left(\frac{x}{\sqrt{x^2 + z^2}} - \ln(-x + \sqrt{x^2 + z^2}) \right) \Big|_{x=a}^{x=b} + \\ & \frac{\phi_0}{(b-c)} \left(\frac{x}{\sqrt{x^2 + z^2}} - \ln(-x + \sqrt{x^2 + z^2}) \right) \Big|_{x=c}^{x=b} \end{aligned} \right) e_z \quad (5)$$

where the parameters a , c , and ϕ_0 are defined Fig. 3. The current circulating around the coil is I , b is the coil radius, n is the number of slots, and z is the depth point. A more general approach to investigate the slotted coil performance should include a current distribution, J_ϕ which is considered uniform over the entire coil area.

2.2 Calculation of the power losses P_A and P_B

The losses can be calculated by integration of the coil current distribution over the area of the coil for P_A , and by integration of the resulting electric field over the volume of the load for P_B . The computation of the power losses was entirely based on the quasi-static method introduced by Ocegueda and Rodriguez [19]. All calculations were moved into Appendixes A and B.

The coil loss P_A , was computed according to coil geometry in Fig. 1c, and it can then be expressed,

$$P_A = \frac{I^2 n}{\sigma_A \delta} \left(\frac{(b+a) \left(\frac{\pi}{n} - \varphi_0 \right)}{b-a} + \frac{2\varphi_0 (b+c)}{b-c} \right) \quad (6)$$

where σ_A is the coil conductivity. The sample loss P_B is

$$P_B = \sigma_B \omega^2 \sum_{m=1}^n \left(\int_{\phi \in \Re_m} \left(\int_0^a |A_{sc}|^2 r^2 dr + \int_a^b |A_{sc}|^2 r^2 dr + \int_b^\infty |A_{sc}|^2 r^2 \sin(\theta) dr \right) \sin(\theta) d\theta d\phi \right. \\ \left. + \int_{\phi \in \Im_m} \left(\int_0^c |A_{cc}|^2 r^2 dr + \int_c^b |A_{cc}|^2 r^2 dr + \int_b^\infty |A_{cc}|^2 r^2 \sin(\theta) dr \right) \sin(\theta) d\theta d\phi \right) \quad (7)$$

where σ_B is the sample conductivity. A_{sc} is the vector potential due to the coil region between slots, and A_{cc} is the vector potential of the slot region.

Once the magnetic field and the power loss expressions were derived, it is possible to calculate a general SNR expression for the slotted surface coil. Then, assuming that the sample resistance is much greater than the coil resistance ($P_B \gg P_A$) [20], Eq. (2) transforms into

$$SNR(z) = \frac{\omega V M B_1(z)}{\sqrt{8kT \Delta f P_B}} \\ = \frac{VM}{\sqrt{8kT \Delta f}} \frac{B_1(z)}{\sqrt{\sigma_B \int_V |A|^2 dV}} \quad (8)$$

combining Eqs. (6) and (8) and replacing them in Eq. (1), Eq. (9) then becomes

$$SNR(z) = \frac{\omega V M \mu_0 I n}{2\pi \sqrt{8kT \Delta f P_B}} \left[\frac{1}{(b-a)} \left(\frac{a\sqrt{b^2+z^2}-b\sqrt{a^2+z^2}}{\sqrt{a^2+z^2}\sqrt{b^2+z^2}} + \ln \left(\frac{-a+\sqrt{a^2+z^2}}{-b+\sqrt{b^2+z^2}} \right) \right) \right. \\ \left. + \frac{1}{(b-c)} \left(\frac{c\sqrt{b^2+z^2}-b\sqrt{c^2+z^2}}{\sqrt{c^2+z^2}\sqrt{b^2+z^2}} + \ln \left(\frac{-c+\sqrt{c^2+z^2}}{-b+\sqrt{b^2+z^2}} \right) \right) \right] \quad (9)$$

Particular SNR formulae can be derived for different coil dimensions to study their behaviour as a function of depth from Eq. (9).

3 Results and Discussion

A general SNR expression of a slotted surface coil was derived using the quasi static approach for n vanes (Eq. 10). This formula was used to theoretically compute *SNR*-vs.-depth profiles to study its behaviour as a function of the depth (z) and various coil design parameters. The profiles of Fig. 4 showed that there is a clear improvement on the performance directly related to the number of vanes for a particular set of parameters.

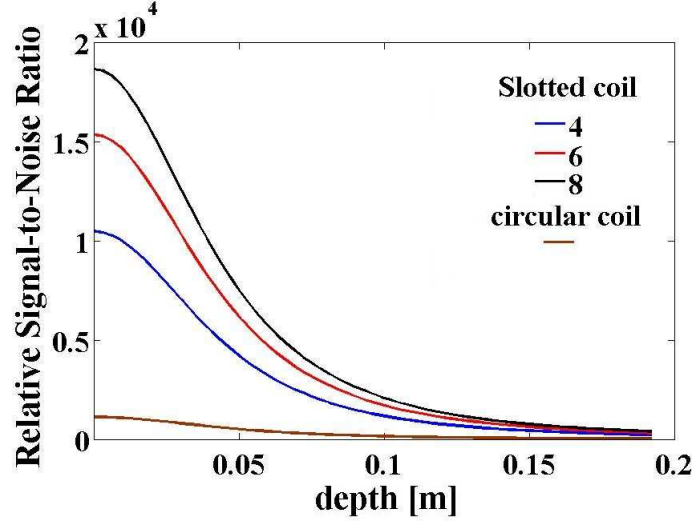


Figure 4. SNR profiles of the slotted surface coil for 4, 6 and 8 vane-type slots and the following design parameters: $a = 2$ cm, b (coil radius) = 6 cm, $c = 4$ cm. All SNR roll-offs were computed using Eq. (10).

As n increases a greater area is covered and the coil configuration may be drastically affected by it. Extra care should be taken since a specific size coil can not accommodate a high number of them. There is a trade off between the number of vane-type slots and their size that it is possible to accommodate in a particular coil size. To avoid a decrement of the *SNR*, coil dimensions should be adjusted to fit more slots in a specific coil size. If a high number of slots is required, a possible coil design candidate for this case may be the coil configuration in Fig. 1b). This is an advantage over other surface coil designs since it actually determines the way to theoretically improve the *SNR* by varying the number of slots and its size.

Additionally, to theoretically compare the slotted coil performance, the *SNR* profile of a circular-shaped coil was also calculated using the quasi-static approach with similar dimensions for fare comparison. There is no specific reason to choose 8 slots, other than following the original design of the cavity magnetron tube, and that the previous experimental results obtained at low frequency (64 MHz). From Fig. 4 the *SNR* stills shows a substantially performance improvement over the *SNR* of a circular coil, for points located perpendicularly further away from the coil plane up to the equivalent distance of the total coil radius. The *SNR* roll-offs theoretically outperformed the circular-shaped coil. Additionally, *SNR* plots as function of the magnetic field intensity were computed and shown in Fig. 5.

These plots exhibit a very similar pattern as that reported in the literature for the low-field scheme. Therefore, the the *SNR* model of slotted coil and the correct selection of the parameters can provide us with reliable guidelines to develop a surface coil with an improved performance. The coil design presented in this work, is only one of the many possible geometries that can be used.

The approach presented here has two important disadvantages, it is only valid for low-field MRI since the quasi-static approach was employed and that a uniform current distribution of the current density was used to derive the *SNR* model. The classical electromagnetic theory poses a great challenge if a more elaborate current distribution is assumed for this coil design. As a first approximation to an intricately difficult problem, an uniform distrubution was assumed despite the fact that this is not accurate enough for a more realistic computation of the magnetic field.

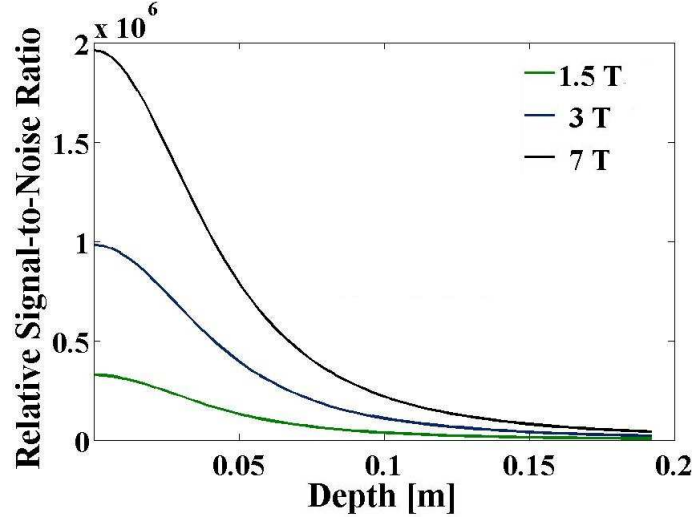


Figure 5. Comparison of theoretically-acquired roll-offs of relative SNR for the slotted surface coil with 8 slots at different magnetic field intensities.

Therefore, this low-field *SNR* model of the slotted surface coil should be viewed as a work in progress whose partial completion has already shown a relatively good agreement with experimental data. It remains then to investigate different approaches to derive other *SNR* models using a high-frequency approach with other coil layouts. However limited this *SNR* model, it can be in different MRI applications such as parallel imaging and phased-array coils, since it allows us to simulate their performance previously to actually start developing coil array prototypes.

This theoretical *SNR* model allows us to simulate the coil performance in advance of experiments thus saving a significant amount of effort in designing these type of coils. This approach to surface receiver coils can be used in MR imaging and MR spectroscopy.

4 Conclusions

The quasi static approach was used to calculate an *SNR* expression for a new coil design called the slotted surface coil. This general *SNR* expression permits to derive particular *SNR* formulae to study the performance of particular coil designs. From the theoretical point of view, the slotted surface coil can produce an important improvement compared with the single surface coil. These theoretical results showed that the slotted surface coil design outperformed the standard circular-shaped coil with similar dimensions. This approach is a good alternative to other schemes used to develop MRI surface coils when dealing with frequency values in the range of the quasi-static approach. This coil design offers a new choice to existing surface coils for different applications in both MRS and MRI.

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Appendix A

Calculation of the power losses P_A

To compute the power loss of the coil due to the Joule heating, P_A , first calculate the average-time rate of work done by the electric field on in the volume V (homogenous conducting half-space, $z > 0$) as follows

$$\frac{dP}{dV} = E \cdot J^* = \frac{1}{\sigma} |J|^2 = \sigma |E|^2 \quad (\text{A.1})$$

Eq. (A.1) was computed using the Poynting's theorem for harmonics fields and Ohm's law. Therefore, the ratio of coil power loss per volume unit is

$$\frac{dP_A}{dV} = \frac{dP_A}{\delta dS'} = \frac{1}{\sigma_A} |J|^2 \quad (\text{A.2})$$

where σ_A is the coil conductivity. From Eq. (A.2) the coil power loss is

$$\begin{aligned} P_A &= \frac{\delta}{\sigma_A} \int_{S'} |J(r')|^2 dS' = \\ &= \frac{\delta}{\sigma_A} \sum_{m=1}^n \int_{\phi' \in \mathfrak{R}_m} J_\phi^2 \rho' d\rho' d\phi' = \frac{I^2 n}{\sigma_A \delta} \left(\frac{(b+a) \left(\frac{\pi}{n} - \varphi_0 \right)}{b-a} + \frac{2\varphi_0 (b+c)}{b-c} \right) \end{aligned} \quad (\text{A.3})$$

Eq. (A.3) transforms into Eq. (3) which is the power loss P_A for the n -slot coil configuration of the slotted surface coil.

Appendix B

Calculation of the power loss P_B

Similarly as in the case of P_A , the ratio of power loss of P_B per volume unit can be written as,

$$\frac{dP_B}{dV} = \sigma_B |E|^2 \quad (\text{B.1})$$

where σ_B is the sample conductivity. To determine the electric field E , the Faraday-Lenz law can be used

$$\nabla \times E = -\frac{\partial B}{\partial t} = j\omega B \quad (\text{B.2})$$

Eq. (B.2) holds true for time variation, $\exp(-j\omega t)$, and the field B can be rewritten $B = \nabla \times A$, so the electric field E transforms into

$$E = -\nabla \Phi + j\omega A \quad (\text{B.3})$$

where $\nabla \Phi$ is the scalar potential and $E = j\omega A$ with A being a vector potential.

The sample power loss using the vector potential, A in Eq. (B.3) becomes

$$P_B = \sigma_B \int_V |E(r)|^2 dV = \sigma_B \int_V |j\omega A|^2 dV = \sigma_B \omega^2 \int_V |A|^2 dV \quad (\text{B.4})$$

where $V = V_1 + V_2$ and the semi-plane $z > 0$. Eq. (B.4) is valid if $\nabla \Phi = 0$ and rewriting A in Cartesian coordinates

$$A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r - r'|} dS' = \frac{\mu_0}{4\pi} \int_{S'} \frac{J_x e_x + J_y e_y}{\left| (x - x')^2 + (y - y')^2 + z^2 \right|^{1/2}} dS' \quad (\text{B.5})$$

To facilitate the computation of the vector potential of Eq. (B.5), the coil surface was split into two potentials corresponding to two different areas: A_{sc} (area surrounding the coil slots) and A_{cc} (slot area). According to the principle of superposition, it is possible to assume that the total coil area can be expressed as $A_{total} = A_{sc} + A_{cc}$, where A_{sc} corresponds to: $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, \dots, \mathfrak{R}_n$.

A_{sc} can be rewritten in spherical coordinates as follows:

$$A_{sc} = \frac{j\phi\mu_0}{4\pi} \sum_{m=1}^n \left(\int_{\phi' \in \mathfrak{R}_m} \int_{r' \in \mathfrak{R}_m} \frac{-r' \sin(\phi')}{\sqrt{r^2 + r'^2 - 2r'r \cos(\gamma)}} dr' d\phi' e_x + \int_{\phi' \in \mathfrak{R}_m} \int_{r' \in \mathfrak{R}_m} \frac{r' \cos(\phi')}{\sqrt{r^2 + r'^2 - 2r'r \cos(\gamma)}} dr' d\phi' e_y \right) \quad (\text{B.6})$$

where $\cos(\gamma) = \sin(\theta) \sin(\phi - \phi')$. A similarly expression can be computed for A_{cc} .

To solve integrals of Eq. (B.6) and the corresponding integral for A_{cc} , we can use the Legendre polynomials, this is a standard procedure used to solve electrostatic problems. Then the denominator of the integrand of (B.4) can be formulated in the frequently-used Legendre polynomials:

$$\frac{1}{\sqrt{r^2 + (r')^2 - 2rr' \cos(\gamma)}} = \begin{cases} \sum_{l=0}^{\infty} \left(\frac{r'}{r^{l+1}} \right) P_l \cos(\gamma) & r < a \\ & a < r' < b \\ \sum_{l=0}^{\infty} \left(\frac{r^l}{r'^{l+1}} \right) P_l \cos(\gamma) & r > b \end{cases} \quad (\text{B.7})$$

$$\left. \begin{aligned} \frac{1}{\sqrt{r^2 + (r')^2 - 2r'r\cos(\gamma)}} &= \sum_{l=0}^{\infty} \left(\frac{r'^l}{r^{l+1}} \right) P_l \cos(\gamma) & a < r' < r \\ &+ \sum_{l=0}^{\infty} \left(\frac{r^l}{r'^{l+1}} \right) P_l \cos(\gamma) & r < r' < b \end{aligned} \right\} a < r < b \quad (\text{B.8})$$

Therefore the vector potential of Eq. B.4 transforms

$$A_{sc} = \frac{J_{\phi} \mu_0}{4\pi} \sum_{l=0}^{\infty} \sum_{m=1}^n \left[\begin{aligned} &\left[\int_a^b \left(\frac{r}{r'} \right)^l dr' + \int_a^r \left(\frac{r'}{r} \right)^{l+1} dr' + \int_r^b \left(\frac{r}{r'} \right)^l dr' + \int_a^b \left(\frac{r'}{r} \right)^{l+1} dr' \right] \\ &\left[- \int_{\phi' \in \mathfrak{R}_m} \sin(\phi') P_l \cos(\gamma) d\phi' e_x + \int_{\phi' \in \mathfrak{R}_m} \cos(\phi') P_l \cos(\gamma) d\phi' e_y \right] \end{aligned} \right] \quad (\text{B.9})$$

In a complete similar fashion and expression for A_{cc} can be derived too.

Finally, the sampe noise P_B becomes

$$P_B = \sigma_B \omega^2 \int_V |A|^2 dV = \sigma_B \omega^2 \left(\int_{V_1} |A_{sc}|^2 dV_1 + \int_{V_2} |A_{cc}|^2 dV_2 \right) \quad (\text{B.10})$$

References

- [1] D. I. Hoult, R. E. Richards, The signal-to-noise ratio of the nuclear magnetic resonance experiment. J. Magn Res. 24 (1976) 71.
- [2] D. I. Hoult, P. C. Lauterbur, The sensitivity of the zeugmatographic experiment involving human samples. J. Magn. Res. 34 (1079) 425.
- [3] J. R. Keltner, J. W. Carlson, M. S. Roos, S. T. S. Wong, T. L. Wong, T. F. Budinger, Electromagnetic fields of surface coil in vivo NMR at high frequencies. Magn. Reson. Med. 22 (1991) 467.
- [4] J. Wang, A. Reykowski, J. Dickas, Calculation of the signal-to-noise ratio for simple surface coils and arrays of coils. IEEE. Trans. Biomed. Eng. 42 (1995) 908.
- [5] W. Schnell, W. Renz, M. Vester, H. Ermert, Ultimate signal-to-noise ratio of surface and body antennas for magnetic resonance imaging. IEEE Trans. Ant. Prop. 48 (2000) 418.
- [6] H. Vesselle, R. E. Collin, The signal-to-noise ratios of nuclear magnetic resonance surface coils and application to a lossy dielectric cylinder model Part I: Theory. IEEE Trans. Biomed. Eng. 42 (1995) 497.
- [7] H. Vesselle, R. E. Collin, The signal-to-noise ratio of nuclear magnetic resonance surface coils and application to a lossy dielectric cylinder model Part II: The case of cylindrical window coils. IEEE Trans. Biomed. Eng. 42 (1995) 507.

- [8] DI Hoult, The principle of reciprocity in signal strength calculations: a mathematical guide. *Concepts Magn. Reson.* 12, (2000) 173.
- [9] T. S. Ibrahim, C Mitchell, P Schmalbrock, R Lee, DW Chakeres, Electromagnetic perspective on the operation of RF coils at 1.5-11.7. Tesla, *Magn Reson Med.* 54 (2005) 683.
- [10] O. Ocali, E. Atalar, Ultimate intrinsic signal-to-noise ratio in MRI. *Magn. Reson. Med.* 39 (1998) 462.
- [11] J. Jin, *Electromagnetic analysis and design in magnetic resonance imaging*. Boca Raton: CRC Press: 1999.
- [12] U. Katscher, P. Börnert, Parallel Magnetic Resonance Imaging. *Top. Magn. Reson. Imaging.* 15 (2004) 127. *Neurotherapeutics.* 4 (2007) 499.
- [13] M. A. Ohlinger, A. K. Grant, D. K. Sodickson, Ultimate intrinsic signal-to-noise ratio for parallel MRI: Electromagnetic Field Considerations. *Magn. Reson. Med.* 50 (2003) 1018.
- [14] F. Wiesinger, P. Boesiger, K. P. Pruessmann. Electrodynamics and ultimate SNR in parallel MR imaging. *Magn. Reson. Med.* 54 (2004) 376.
- [15] R. V. Pound, From radar to nuclear magnetic resonance. *Rev. Mod. Phys.* 71 (1999) S54.
- [16] B. Collins. The Magnetron. In: L. N. Ridenour, ed. *Radar System Engineering*. New York; McGraw-Hill: 1947. Chap. 10.
- [17] A. O. Rodriguez, Magnetron coil for brain MR imaging. *Arch. Med. Res.* 37 (2006) 804.
- [18] J. D. Jackson, *Classical electrodynamics*. 3rd ed. New York; Wiley & Sons: 1999. pp. 181-184.
- [19] K. Ocegueda, A. O. Rodriguez, A simple method to calculate the signal-to-noise ratio of a circular-shaped coil for MRI. *Conc. Magn. Reson. Part A*, 28A (2006) 422.
- [20] W. A. Edelstein, G. H. Glover, C. J. Hardy, R. W. Redington, The intrinsic signal-to-noise ratio in NMR imaging. *Magn. Reson. Med.* 3 (1986) 604.